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Decentralized Sliding Mode Control in Three-Axis Inertial Platforms

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The accuracy improvement of three-axis inertial platform stabilization systems with a dynamically tuned gyroscope sensor is addressed by methods of variable structure control. The decentralized sliding mode control strategy has been worked out and applied for the design of the stabilization system of the three-axis inertial platform. The nonlinear transformations of bases in the nonlinear interconnected subsystems have been found, based on remaining dynamics. These transformations changed the subsystems to the forms convenient for the sliding mode synthesis. The local sliding manifolds have been designed in the form of nonlinear functions and nonlinear dynamic operators to provide the desired linear decoupled output tracking (stabilization) in each axis of the inertial platform stabilization system. The system with the dynamic sliding mode controller obtained the combined features of the system with a conventional dynamic compensator (accommodation to unmatched disturbances) and a conventional sliding mode controller (insensitivity to matched disturbances). The results of the simulation of the three-axis inertial platform stabilization system with the designed conventional and the dynamic sliding mode controllers showed significant improvement of the accuracy (2–3 times better then in the systems with linear controllers) of the platform stabilization.

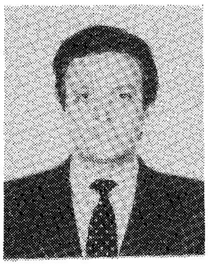
Introduction

ACCURACY improvement of a three-axis inertial platform stabilization system[1] with "passengers" (accelerometers and other devices serving as sensors for wing and ballistic missiles) is considered in this paper. This accuracy is closely related to the accuracy of the data, obtained from the passengers, and, thus far, to the accuracy of the overall performance of a missile. The stabilization of inertial platforms on sliding modes was partially studied and solved by the author of this paper in 1985–1990 at the Department of Applied Mathematics of Chelyabinsk Technical University, Chelyabinsk, Russia. Some of the sliding mode control algorithms have been realized in inertial platform stabilization systems of various Russian wing missiles.

Three-axis inertial platform stabilization systems are usually described by three nonlinear interconnected subsystems of differential equations with disturbances and plant uncertainties.[1] The objective of decentralized sliding mode control in such systems is to force the interconnected stabilization systems of all of the axes to move to the local manifolds (sliding surfaces) and to maintain these systems in sliding manifolds thereafter by high-speed switching control functions.[2–4] The local sliding manifolds (surfaces) have to be chosen such that the inertial platform output stabilization error performances are desired in these surfaces.

We think of the desired performance of the output stabilization error in each axis of a three-axis inertial platform stabilization system as decoupled and linear with desired response characteristics. For example, the performance of the output stabilization errors would be desired if these errors are the outputs of decoupled systems of linear time-invariant differential equations with given eigenvalues placement of the matrices of these systems.

The latest results in variable structure systems in nonlinear tracking via nonlinear sliding manifolds have been obtained based on a nonlinear inversion of the remaining dynamics[5] and a representation of the system in a global generalized controller canonical form[6] combined with linearization technique by nonlinear feedback.[7] These concepts were realized in various nonlinear control systems including continuous aircraft trajectory control[8] and one-input-one-output nonlinear dynamic sliding mode output tracking.[9,10] In the last two works the concept of dynamic sliding surface was naturally introduced based on the equations of nonlinear inverse dynamics. The resulting dynamic sliding mode controller includes a conventional part and a dynamic compensator, which postmultiplied the switching element. It gives extra opportunities in achieving "smoothness" in output tracking. The concept of the dynamic sliding surface as a linear dynamic compensator specified in time domain and placed before the switching element was introduced in the Ref. 11. Such a dynamic sliding mode controller also allows a lot of flexibility in the "frequency shaping" of the sliding mode. The dynamic sliding controller with the combined features of a conventional dynamic compensator (accommodation to unmatched disturbances) and a conventional sliding mode controller (insensitivity to matched disturbances) was introduced in Ref. 12. Applications of variable structure systems (sliding modes) in aircraft and missile control were presented in Refs. 13 and 14.



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It is obvious that nonlinear conventional and dynamic sliding manifolds offer a richer variety of controller designs compared to traditional linear surfaces. In this paper we employ the concept of the inverse dynamics[7,8] for a design of decentralized conventional nonlinear sliding manifolds[4] and decentralized sliding manifolds as nonlinear dynamic operators.[9–12] This concept is realized in a sliding mode control output tracking with the application to the solution of the three-axis inertial platform stabilization problem.

As a first step for the sliding mode controller design, we identify the nonlinear basis transformation in the nonlinear interconnected subsystems, based on the system inverse dynamics, to change the whole system to a form convenient for the local sliding mode synthesis. As a second step, we design the nonlinear local sliding manifolds as dynamic operators[12] and the nonlinear conventional local sliding manifolds[2–4] to meet the requirements of the performance of the subsystem output tracking errors in the sliding mode. For the third step, the discontinuous local control laws should be designed to provide the existence of the designed sliding modes. This procedure has been applied to the sliding mode controller design in the stabilization system of a three-axis inertial platform. The results of the design and simulation of this system are discussed in the paper.

The contribution of this paper is 1) the development of the design of a decentralized sliding mode controller based on the local nonlinear conventional and the local nonlinear dynamic sliding manifolds for the sliding mode control tracking (stabilization) and 2) the solution of the stabilization problem of a three-axis inertial platform with a dynamically tuned gyroscope sensor via a decentralized sliding mode controller design based on the nonlinear conventional and dynamic sliding manifolds.

Decentralized Sliding Mode Control Output Tracking Problem

A nonlinear system consisting of three interconnected subsystems is described as follows:

$$\begin{aligned} \dot{\boldsymbol{x}}_k &= \boldsymbol{A}_k\boldsymbol{x}_k + \boldsymbol{F}_k(t, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) + \boldsymbol{B}_k u_k \\ y_k &= \boldsymbol{G}_k \boldsymbol{x}_k \end{aligned} \tag{1}$$

where $k(= 1, 2, 3)$ is the number of the subsystem, $\boldsymbol{x}_k(\in \boldsymbol{R}^{n_k})$ is the state vector, $u_k(\in \boldsymbol{R}^1)$ is the control function, and $y_k(\in \boldsymbol{R}^1)$ is the measuring accessible output. $\boldsymbol{A}_k$, $\boldsymbol{B}_k$, and $\boldsymbol{G}_k$ are constant matrices of appropriate dimensions, and $\boldsymbol{F}_k(t, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)(\in \boldsymbol{R}^{n_k})$ are partially known unmatched vector functions (internal and interaction disturbances, plant uncertainties, and nonlinearities). $\{\boldsymbol{A}_k, \boldsymbol{B}_k\}$ are controllable, and $y_k^*(t)$ is given in current time reference output trajectories (constants in the case of a stabilization) $\forall k = 1, 2, 3$.

Remark 1. The vector-function $\boldsymbol{F}_k(t, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$ is said to be unmatched[2,3] if a scalar function $\lambda_k(t, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$ does not exist to meet the following equality: $\boldsymbol{B}_k\lambda_k(t, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) = \boldsymbol{F}_k(t, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$.

The objective is to design the control functions $u_k \forall k = 1, 2, 3$ to provide the desired linear decoupled output tracking: $\lim\|e_k(t)\| = 0$ as $t \to \infty$ and to have the ability to control the rates of these limits, where $e_k = y_k^* - y_k \forall k = 1, 2, 3$.

We will look for the solution of this problem in decentralized sliding modes. This is to find the switching control functions u_k $\forall k = 1, 2, 3$ in the form

$$u_k = \begin{cases} u_k^+, & \Phi_k(\boldsymbol{x}, \boldsymbol{y}^*, t) > 0 \\ u_k^-, & \Phi_k(\boldsymbol{x}, \boldsymbol{y}^*, t) < 0 \end{cases} \tag{2}$$

where

$$\Phi_k(\boldsymbol{x}, \boldsymbol{y}^*, t) = \sigma_k(\boldsymbol{x}, \boldsymbol{y}^*, t) = 0$$

are the conventional sliding manifolds or

$$\Phi_k(\boldsymbol{x}, \boldsymbol{y}^*, t) = \Im_k(\boldsymbol{x}, \boldsymbol{y}^*, t) = 0$$

are the sliding manifolds as the dynamic operators, and u_k^+ and u_k^- are continuous functions of t and x;

$$\boldsymbol{x} = \left\{\boldsymbol{x}_1^T, \boldsymbol{x}_2^T, \boldsymbol{x}_3^T\right\}^T, \qquad \boldsymbol{y}^* = \left\{(y_1^*)^T, (y_2^*)^T, (y_3^*)^T\right\}^T \forall k = 1, 2, 3$$

The desired linear decoupled output tracking should be provided in the correspondingly designed sliding manifolds $\Phi_k(\boldsymbol{x}, \boldsymbol{y}^*, t) = 0$ $\forall k = 1, 2, 3$. The existence of these sliding modes (the motion in the sliding manifolds) will be guaranteed by the corresponding design of the continuous control functions u_k^+ and u_k^- $\forall k = 1, 2, 3$.

It would be ineffective to introduce the sliding modes directly into the system (1). The problem is that the unmatched nonlinear disturbances $F_k(t, x)$ could reduce the tracking accuracy and destroy the decoupling of the subsystems by effecting the output tracking errors e_k in the sliding modes.[2–4] In this case the control objective will not be achieved. As a first step of the sliding mode controller design we will identify the nonlinear basis transformations in the nonlinear interconnected subsystems, based on the system of inverse dynamics, to change the whole system to a form convenient for the local sliding mode synthesis.

Remark 2. We will think of the system (1) to be in a convenient form if its tracking errors e_k are the outputs of the nonlinear interconnected subsystems of the differential equations with matched disturbances, interconnection, and nonlinear terms.

Nonlinear Transformation of the Basis

The main result of this section is formulated in the following lemma.

Lemma. System (1) in the new basis

$$\begin{bmatrix} z_k^1 \\ z_k^2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_k^{1*} \\ x_k^{2*} \end{bmatrix} - \begin{bmatrix} \boldsymbol{M}_k^1 \\ \boldsymbol{M}_k^2 \end{bmatrix} \boldsymbol{x}_k \qquad \forall k = 1, 2, 3 \tag{3}$$

has the following convenient form:

$$\dot{z}_k^1 = \boldsymbol{A}_k^{11} z_k^1 + \boldsymbol{A}_k^{12} z_k^2$$

$$\begin{aligned} \dot{z}_k^2 &= \boldsymbol{A}_k^{21} z_k^1 + A_k^{22} z_k^2 - B_k^2 u_k \\ &\quad + \left[\dot{x}_k^{2*} - \boldsymbol{A}_k^{21}\boldsymbol{x}_k^{1*} - A_k^{22} x_k^{2*} - F_k^2(\boldsymbol{x}, t)\right] \end{aligned} \tag{4}$$

$$e_k = \boldsymbol{G}_k^1 z_k^1 + G_k^2 z_k^2; \quad z_k^1 \in R^{n_k - 1};\ z_k^2, e_k \in R^1$$

where $\boldsymbol{x}_k^{1*}$ and x_k^{2*} are defined through the nonlinear descriptor system[15] as

$$\begin{bmatrix} \boldsymbol{I}_{(n_k-1)*(n_k-1)} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{x}}_k^{1*} \\ \dot{x}_k^{2*} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_k^{11} & \boldsymbol{A}_k^{12} \\ \boldsymbol{G}_k^1 & G_k^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_k^{1*} \\ x_k^{2*} \end{bmatrix} + \begin{bmatrix} \boldsymbol{F}_k^1(\boldsymbol{x}, t) \\ -y_k^*(t) \end{bmatrix} \tag{5}$$

and the nonsingular linear transformations $\boldsymbol{M}_k = \begin{bmatrix} M_k^1 \\ M_k^2 \end{bmatrix}$ have been specified as follows[2,16,17]:

$$\boldsymbol{M}_k\boldsymbol{B}_k = \begin{bmatrix} \boldsymbol{M}_k^1\boldsymbol{B}_k \\ \boldsymbol{M}_k^2\boldsymbol{B}_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ B_k^2 \end{bmatrix}, \qquad B_k^2 \neq 0 \tag{6a}$$

$$\boldsymbol{M}_k\boldsymbol{A}_k\boldsymbol{M}_k^{-1} = \begin{bmatrix} \boldsymbol{A}_k^{11} & A_k^{12} \\ \boldsymbol{A}_k^{21} & A_k^{22} \end{bmatrix} \tag{6b}$$

$$\begin{bmatrix} \boldsymbol{M}_k^1 \\ \boldsymbol{M}_k^2 \end{bmatrix} \boldsymbol{F}_k = \begin{bmatrix} \boldsymbol{F}_k^1 \\ F_k^2 \end{bmatrix}, \qquad \boldsymbol{G}_k \begin{bmatrix} \boldsymbol{M}_k^1 \\ \boldsymbol{M}_k^2 \end{bmatrix}^{-1} = \left[\boldsymbol{G}_k^1 \vdots G_k^2\right] \tag{6c}$$

Proof. Let us assume that the "reference" state trajectories $\begin{bmatrix} x_k^{1*} \\ x_k^{2*} \end{bmatrix}$ are given and the transformation matrices $\boldsymbol{M}_k = \begin{bmatrix} M_k^1 \\ M_k^2 \end{bmatrix}$ are calculated following formulas (6) $\forall k = 1, 2, 3$. Then system (1) can be rewritten in the new basis equation (3):

$$
\begin{aligned}
\dot{z}_k^1 &= A_k^{11} z_k^1 + A_k^{12} z_k^2 + \left[\dot{x}_k^{1*} - A_k^{11} x_k^{1*} - A_k^{12} x_k^{2*} - F_k^1(x, t)\right] \\
\dot{z}_k^2 &= A_k^{21} z_k^1 + A_k^{22} z_k^2 - B_k^2 u \\
&\quad + \left[\dot{x}_k^{2*} - A_k^{21} x_k^{1*} - A_k^{22} x_k^{2*} - F_k^2(x, t)\right] \qquad (7)\\
e_k &= G_k^1 z_k^1 + G_k^2 z_k^2 + \left[y_k^* - G_k^1 x_k^{1*} - G_k^2 x_k^{2*}\right]
\end{aligned}
$$

Let us assume now that the reference state trajectories $\begin{bmatrix} x_k^{1*} \\ x_k^{2*} \end{bmatrix}$ satisfy the descriptor system (5) $\forall k = 1, 2, 3$. It means that

$$
\begin{aligned}
\dot{x}_k^{1*} - A_k^{11} x_k^{1*} - A_k^{12} x_k^{2*} - F_k^1(x, t) &= 0 \\
y_k^* - G_k^1 x_k^{1*} - G_k^2 x_k^{2*} &= 0
\end{aligned}
$$

Consequently, the system (7) has been transformed to the system (4). The lemma is proved. □

Remark 3. The descriptor system (5) [or the system of nonlinear differential-algebraic equations (DAE)[15]] looks like some sort of the inverse dynamics[7–9] of system (1) and is called the system center.[17,18]

Remark 4. We need to identify the matrices M_k and the reference state trajectories x_k^{1*} and x_k^{2*} $\forall k = 1, 2, 3$ to transform system (1) to a convenient form, Eq. (4). There is no problem to specify the transformation matrices M_k in Eq. (6a). The problem is to solve the nonlinear system of the differential algebraic equation (5) to identify the reference state trajectories x_k^{1*} and x_k^{2*} $\forall k = 1, 2, 3$. Solving this system we employ the algorithm of the DAE solution based on differentiation of the algebraic constrains.[19] To find the solution we need to estimate the reference profiles $y_k^*(t)$, the unmatched parts $F_k^1(t, x)$ of the nonlinear terms $F_k(t, x)$ $\forall k = 1, 2, 3$, and probably their derivatives.[15,19] Usually it is no problem to estimate the reference profiles $y_k^*(t)$. However, estimation of $F_k^1(t, x)$ and its derivatives is not easy. The advantage we have is that we need to estimate just the unmatched parts of $F_k(t, x)$. This is simpler than to estimate the whole function $F_k(t, x)$, applying some feedback linearization control algorithms[5,7] to the solution of the decentralized nonlinear output tracking problem. We also need to specify the initial conditions for x_k^{1*} and x_k^{2*} $\forall k = 1, 2, 3$ to identify the unique solution of the differential algebraic equation (5). It is not easy. However, solving a system of DAE we do not need to know all initial conditions.[15] There are cases when we do not need to have the initial conditions at all.[15,19] Another problem is the stability analysis of the system of DAE. The issue is that the solution of x_k^{1*} and x_k^{2*} $\forall k = 1, 2, 3$ has to be stable. In the opposite case, the sliding mode in the system could not be operational. This problem is similar to the inverse dynamics stability problem.[5,7] The solution of the system of differential algebraic equation (5) will be given later during the three-axis inertial platform stabilization system synthesis.

Decentralized Nonlinear Sliding Manifold Design

Let us assume that the system (1) is transformed to a convenient form equation (4) at the first step of a sliding mode controller design. Now, we could start the second step. This is to design the nonlinear local sliding manifolds as dynamic operators[9–12] and the nonlinear conventional local sliding manifolds[2–4,16] to meet the requirements of the performance of the subsystem output tracking errors in the decentralized sliding mode.

Let us consider the local sliding manifold as linear dynamic operators[11,12] acting on the output errors $e_k = y_k^* - y_k$. They are

$$\Im_k = z_k^2 + W_k(s) e_k = 0, \ \forall k = 1, 2, 3$$

where

$$\Im_k, z_k^2, e_k \in R^1, \qquad s = \frac{\mathrm{d}}{\mathrm{d}t}, \qquad W_k(s) = \frac{P_m^k(s)}{Q_n^k(s)} \tag{8}$$

and

$$Q_n^k(s), \text{ and } P_m^k(s)$$

are polynomials of s and $m \leq n$. The following theorem has been formulated and proved.

Theorem 1. Suppose that the sliding mode exists in the system (4) in local dynamic sliding manifolds Eq. (8). Then the output tracking errors $e_k = y_k^* - y_k$ in the system (4) are described by the decoupled linear operator equations

$$
\left[1 + G_k^2 W_k(s) + G_k^1 \left(Is - A_k^{11}\right)^{-1} A_k^{12} W_k(s)\right] e_k = 0 \qquad \forall k = 1, 2, 3 \tag{9}
$$

in the sliding manifold equation (8). □

Proof. Concerning the concept of equivalent control[2,3,16] and the concept of a dynamic sliding manifold[11,12] the system (4) in the dynamic sliding manifold equation (8) becomes

$$
\begin{aligned}
\dot{z}_k^1 &= A_k^{11} z_k^1 + A_k^{12} z_k^2, \qquad & z_k^2 &= -W(s) e_k \\
e_k &= G_k^1 z_k^1 + G_k^2 z_k^2, \qquad & \forall k &= 1, 2, 3
\end{aligned} \tag{10}
$$

Equations (10) can be rewritten in operator notation as follows:

$$
\begin{aligned}
sIz_k^1 &= A_k^{11} z_k^1 - A_k^{12} W_k(s) e_k \\
e_k &= G_k^1 z^1 - G_k^2 W_k(s) e_k
\end{aligned} \qquad \forall k = 1, 2, 3 \tag{11}
$$

Solving Eqs. (11) for e_k we have operator equations (9) immediately. Theorem 1 is proved. □

Theorem 1 can be applied to the dynamic sliding manifold design as follows. Specify the desired dynamic operators

$$
H_k^*(s) = 1 + W_k(s) G_k^2 + G_k^1 \left(Is - A_k^{11}\right)^{-1} A_k^{12} W_k(s) \qquad \forall k = 1, 2, 3 \tag{12}
$$

Find the corresponding dynamic operators

$$
W_k(s) = \left[G_k^2 + G_k^1 \left(Is - A_k^{11}\right)^{-1} A_k^{12}\right]^{-1} [H_k^*(s) - 1] \qquad \forall k = 1, 2, 3 \tag{13}
$$

independently for each subsystem. Design the dynamic sliding manifolds using formulas (8) transformed to the initial basis of the system (1). This is

$$\Im_k = \Im_k^{\text{glob}} + \Im_k^{\text{loc}} = 0 \qquad \forall k = 1, 2, 3 \tag{14}$$

where

$$\Im_k^{\text{glob}} = x_k^{2*}, \qquad \Im_k^{\text{loc}} = W_k(s) \cdot (y_k^* - y_k) - M_k^2 x_k$$

It is obvious that each $\Im_k^{\text{glob}}$ depends nonlinearly on x, t, $y_k^*(t)$ through the nonlinear system of differential algebraic equation (5) and can be considered as a nonlinear global entry of the dynamic sliding manifold equation (14). On the other hand $\Im_k^{\text{loc}}$ depends only on local variables x_k, y_k^*, and y_k and we can think about $\Im_k^{\text{loc}}$ as the local entry of the dynamic sliding manifold equation (14). Consequently, we can consider the sliding manifold equation (14) as the decentralized nonlinear dynamic sliding manifolds.

The conditions for the conventional local sliding manifolds design in the decentralized nonlinear output tracking can be derived from the conditions of the Theorem 1. This particular case of the dynamic sliding manifold design has been formulated in the following corollary.

Corollary 1. Suppose that the sliding mode exists in the system (4) in local dynamic sliding manifold equation (8), and the dynamic operators $W_k(s) = [P_m^k(s)/Q_n^k(s)]$ are selected to satisfy the equalities

$$\left(1 + W_k(s) G_k^2\right)^{-1} W_k(s) G_k^1 z_k^1 = c_k z_k^1 \tag{15}$$

where $c_k \in R^{n_k - 1}$ are constant row vectors $\forall k = 1, 2, 3$. Then the dynamic sliding manifold equation (8) become the conventional sliding manifolds

$$\sigma_k = z_k^2 + c_k z_k^1 = 0 \qquad \forall k = 1, 2, 3 \tag{16}$$

and the output tracking errors $e_k = y_k^* - y_k$ in system (4) are described by the decoupled linear time-invariant systems of differential equations

$$\dot{z}_k^1 = \left(A_k^{11} - A_k^{12}c_k\right)z_k^1, \qquad z_k^2 = -c_k z_k^1$$
$$e_k = \left(G_k^1 - G_k^2 c_k\right)z_k^1 \qquad \forall k = 1, 2, 3 \tag{17}$$

Proof. Substituting the output of the system (4) in expression (8) we acquire the following formula for the dynamic sliding manifold:

$$\Im_k = \left[1 + W_k(s)G_k^2\right]z_k^2 + W_k(s)G_k^1 z_k^1 = 0 \tag{18}$$

Using the property that the equations of the sliding mode should not be changed if the equation of the sliding manifold is to be premultiplied by the nonsingular matrix,[2,3] we transform formulas (18) as follows:

$$\tilde{\Im}_k = \left[1 + W_k(s)G_k^2\right]^{-1}\Im_k = z_k^2 + \left[1 + W_k(s)G_k^2\right]^{-1} \times W_k(s)\,G_k^1 z_k^1 = 0 \tag{19}$$

Let us suppose that the dynamic operators $W_k(s) = [P_m^k(s)/Q_n^k(s)]$ are selected to satisfy equalities (15). Then the dynamic sliding manifold equation (19) become the conventional ones $\forall k = 1, 2, 3$. They are

$$\sigma_k = z_k^2 + c_k z_k^1 = 0 \tag{20}$$

where $c_k \in (\boldsymbol{R}^{n-1})$ are constant row vectors. Suppose that the sliding mode exists in system (4) in the dynamic sliding manifolds Eq. (8). Then the sliding mode will exist in the particular case of these manifolds, Eq. (20). Applying the concept of equivalent control[2,3,16] we can immediately identify the description of the system (4) sliding mode in the conventional sliding manifold equation (20) in the form of Eqs. (17). The corollary is proved. □

Corollary 1 can be applied to the conventional sliding manifold design as follows. Identify vectors c_k to provide the desired eigenvalues placement of the matrices of the systems (17) $(A_k^{11} - A_k^{12}c_k)$ (Refs. 2, 3, and 16) independently for each subsystem. Then design the conventional sliding manifold equation (20) transformed to the initial basis of system (1). This is

$$\sigma_k = \sigma_k^{\text{glob}} + \sigma_k^{\text{loc}} \qquad \forall k = 1, 2, 3 \tag{21}$$

where

$$\sigma_k^{\text{glob}} = [c_k, 1]\begin{bmatrix} x_k^{1*} \\ x_k^{2*} \end{bmatrix}, \qquad \sigma_k^{\text{loc}} = -[c_k, 1]M_k x_k$$

It is obvious that each σ_k^{glob} depends nonlinearly on x, t, $y_k^*(t)$ through the nonlinear system of the differential algebraic equation (5) and can be considered as a nonlinear global entry of the conventional sliding manifold Eq. (21). On the other hand σ_k^{loc} depends only on local variables x_k. Thus, we can think about σ_k^{loc} as the local entry of the conventional sliding manifold equation (21). Consequently, we can think about the sliding manifold equation (21) as the decentralized conventional sliding manifolds.

Local Discontinuous Control Function Design

Let us suppose that the local dynamic nonlinear sliding manifold equations (8) or (14) have been designed following Theorem 1 at the second step of the sliding mode controller design. Now we could start the third step. This is to specify the corresponding discontinuous local control laws (2) to provide the existence of the sliding mode in the system (4) in the designed sliding manifold equations (8) or (14). The following theorem has been formulated and proved:

Theorem 2. The sliding mode in the system (4) exists in the local dynamic sliding manifold equations (8) or (14) if the discontinuous control laws (2) are chosen as

$$u_k = u_{k\text{eq}} + \rho_k\left(B_k^2\right)^{-1}\left[1 + W_k(s)G_k^2\right]^{-1} \cdot \text{sign}(\Im_k) \tag{22}$$

where $\rho_k > 0$ are arbitrary chosen constants and $u_{k\text{eq}}$ are the equivalent control functions[2,3]

$$u_{\text{eq}} = \left(B_k^2\right)^{-1} \cdot \left[1 + W_k(s)G_k^2\right]^{-1}\left\{\left[A_k^{21} + W_k(s)G_k^1 A_k^{11} + W_k(s)G_k^2 A_k^{21}\right]z_k^1 + \left[A_k^{22} + W_k(s)G_k^1 A_k^{12} + W_k(s)G_k^2 A_k^{22}\right]z_k^2 + \left[1 + W_k(s)G_k^2\right]\Phi_k\right\} \tag{23}$$

with functions

$$\Phi_k = \dot{x}_k^{*2} - A_k^{21}x_k^{*1} - A_k^{22}x_k^{*2} - F_k^2(t, x_1, x_2, x_3) \tag{24}$$

Proof. The following candidate for the Lyapunov function has been built:

$$V = \frac{1}{2}\sum_{k=1}^{3}(\Im_k)^2 > 0 \tag{25}$$

where

$$\Im_k = z_k^2 + W_k(s)e_k \qquad \forall k = 1, 2, 3$$

are specified following expression (8). The derivative of the candidate to the Lyapunov function (25) has been found on the states of the system (4) as follows:

$$\dot{V} = \sum_{k=1}^{3}\Im_k\dot{\Im}_k = \sum_{k=1}^{3}\Im_k\left\{\left[A_k^{21} + W_k(s)G_k^1 A_k^{11} + W_k(s)G_k^2 A_k^{21}\right]z_k^1 + \left[A_k^{22} + W_k(s)G_k^1 A_k^{12} + W_k(s)G_k^2 A_k^{22}\right]z_k^2 + \left[1 + W_k(s)G_k^2\right]\Phi_k - \left[1 + W_k(s)G_k^2\right]B_k^2 u_k\right\} \tag{26}$$

Substitution of formula (23) into expression (26) yields the final equation for the derivative of the candidate to the Lyapunov function. This is

$$\dot{V} = -\sum_{k=1}^{3}\rho_k \cdot \Im_k \cdot \text{sign}(\Im_k) = -\sum_{k=1}^{3}\rho_k \cdot |\Im_k| < 0$$
$$\forall \rho_k > 0, k = 1, 2, 3$$

It means that the manifold $\Im_k = 0\ \forall k = 1, 2, 3$ is attractive (or equilibrium point $\Im_k = 0\ \forall k = 1, 2, 3$ is stable), and system (4) sliding mode exists in the local dynamic sliding manifold equation (8). The theorem 2 is proved. □

The existence conditions of the sliding mode in system (4) in the conventional local sliding manifold equation (16) can be derived from the conditions of the Theorem 2. This particular case of the dynamic sliding manifold existence (stability) has been formulated in the following corollary.

Corollary 2. Suppose that the dynamic operators $W_k(s) = [P_m^k(s)/Q_n^k(s)]$ are selected to satisfy equalities (15). Then the sliding mode in system (4) exists in the local conventional sliding manifold equation (16) if the discontinuous control laws (2) are chosen as follows:

$$u_k = u_{k\text{eq}} + \rho_k\left(B_k^2\right)^{-1}\text{sign}(\sigma_k) \tag{27}$$

where $\rho_k > 0$ are arbitrary chosen constants, and $u_{k\text{eq}}$ are the equivalent control functions[2,3]

$$u_{k\text{eq}} = \left(B_k^2\right)^{-1}\left[\left(A_k^{21} + c_k A_k^{11}\right)z_k^1 + \left(A_k^{22} + c_k A_k^{12}\right)z_k^2 + \Phi_k\right] \tag{28}$$

with functions Φ_k defined by expression (24).

The proof of Corollary 2 is similar to the proof of Corollary 1.

Three-Axis Inertial Platform Stabilization System Design

The kinematics scheme of a three-axis inertial platform stabilization system with dynamically tuned gyroscope sensors is demonstrated[1] in Fig. 1, where 1–3 indicate the gimbals, 4 the inertial platform, 5 and 6 the dynamically tuned gyroscopes, 7 the amplifier, 8 the transformer of coordinates, 9–12 the torque servo motors, 13 the angle transformer, and 14–16 the tachometers.

The following simplified description of a sensor based on dynamically tuned gyroscope is accepted[1]:

$$W_g(s) = \frac{V_g(s)}{\alpha(s)} = k_g \frac{1 + 2\xi_1 T_1 s + T_1^2 s^2}{1 + 2\xi_2 T_2 s + T_2^2 s^2} \tag{29}$$

where $V_g(s)$ and $\alpha(s)$ are the Laplace transforms of the output voltage of a sensor $v_g(t)$ and the measured position angle of one of the platform axes $\alpha(t)$, respectively; $\xi_1, \xi_2 \ll 1$ and $T_1 \approx T_2$. In our case $\xi_1 = 2 \times 10^{-3}$, $\xi_2 = 10^{-3}$, $T_1 = 5.3 \times 10^{-4}$, $T_2 = 5.2 \times 10^{-4}$.

The simplified nonlinear description of the x axis of the three axes inertial platform with the dynamically tuned gyroscope sensor Eq. (29) is considered.[1,12] This is

$$\begin{aligned}
\frac{\mathrm{d}i^x}{\mathrm{d}t} &= -\frac{1}{T^x} i^x - \frac{k_b^x}{L^x}\Omega^x + \delta^x(i^x, \Omega^x) + \frac{1}{L^x}u^x \\
\frac{\mathrm{d}\Omega^x}{\mathrm{d}t} &= \frac{k_t^x}{J^x} i^x - \frac{b^x}{J^x}\Omega^x - \frac{1}{J^x}\left(T_L^x + T_L^{yz}\right) \\
\frac{\mathrm{d}\alpha^x}{\mathrm{d}t} &= \Omega^x \\
\frac{\mathrm{d}v_g^x}{\mathrm{d}t} &= r^x \\
\frac{\mathrm{d}r^x}{\mathrm{d}t} &= \frac{1}{T_2^2}\left[\left(k_g^x + \frac{T_1^2}{J^x}\right) i^x + \left(2\xi_1 + \frac{T_1^2}{J^x}\right)\Omega^x + k_g^x \alpha^x - v_g^x \right. \\
&\quad \left. - 2\xi_2 T_2 r^x + \frac{T_1^2}{J^x}\left(T_L^x + T_L^{yz}\right)\right] \\
y^x &= v_g^x
\end{aligned} \tag{30}$$

where

$$T_L^{yz} = P_1^x \Omega^y + P_2^x \Omega^z + P_3^x \Omega^y \Omega^z$$

is the disturbance torque to the x axis from the y and z axes, indices x, y, and z correspond to the x, y, and z axes, i^x is current in the armature of a torque motor, and α^x is a position angle of the inertial platform. Ω^x, Ω^y, and Ω^z angular frequencies of the position angles of the inertial platform, v_g^x is an output voltage of a sensor; r^x is an internal state coordinate of the dynamically tuned gyroscope, $\delta^x(i^x, \Omega^x)$ are bounded uncertainties and nonlinearities of a plant, u^x is voltage applied to the armature of a torque motor (a control function). Also, T_L^x, T_L^y, and T_L^z bounded load torque (disturbances), J^x is the moment of inertia of the platform with a gyroscope sensor about the x axis, and $y^{x*}(t) \equiv 0$ is the reference value of a position angle in volts.

Remark 5. It is clear that external and interaction disturbances T_L^x and T_L^{yz} are unmatched.

Remark 6. To have the description of three-axis inertial platform one has to build two more systems of differential equations substituting y for x (for y axis) and z for x (for z axis) in Eqs. (30). These three interconnected systems of the differential equations give us the description of the three-axis inertial platform.

The following values of the parameters of system (30) are given:

$$\begin{gathered}
L_a^x = L_a^y = L_a^z = 0.37 \text{ H} \\
k_t^x = k_t^y = k_t^z = 7.5 \times 10^{-2} \text{ Nm/A} \\
k_g^x = k_g^y = k_g^z = 10^3 \text{ V/rad} \\
k_b^x = k_b^y = k_b^z = 5 \text{ Vs/rad} \\
b^x = b^y = b^z = 0.28 \times 10^{-2} \text{ Nms} \\
J^x = J^y = 0.4 \times 10^{-2} \text{ kg-m}^2 \\
\delta^k(i^k, \Omega^k) = i^k \cdot \sin 1884t \text{ A/s}, \quad k = x, y, z \\
J^z = 0.15 \times 10^{-2} \text{ kg-m}^2 \\
\omega^x = \omega^y = \omega^z = 1884 \text{ rad/s} \\
A^x = A^y = A^z = 5 \cdot 10^{-3} \text{ V}, \qquad P_1^x = P_1^y = 0.4 \text{ kg-m}^2\text{/s} \\
P_2^x = P_2^y = 0.24 \text{ kg-m}^2\text{/s}, \qquad P_3^x = P_3^y = -20 \text{ kg-m}^2 \\
P_1^z = 0.3 \text{ kg-m}^2\text{/s}, \quad P_2^z = 0.15 \text{ kg-m}^2\text{/s}, \quad P_3^z = -15 \text{ kg-m}^2
\end{gathered} \tag{31}$$

The desired performance indices of the stabilization systems in x, y, and z axes are specified as follows.

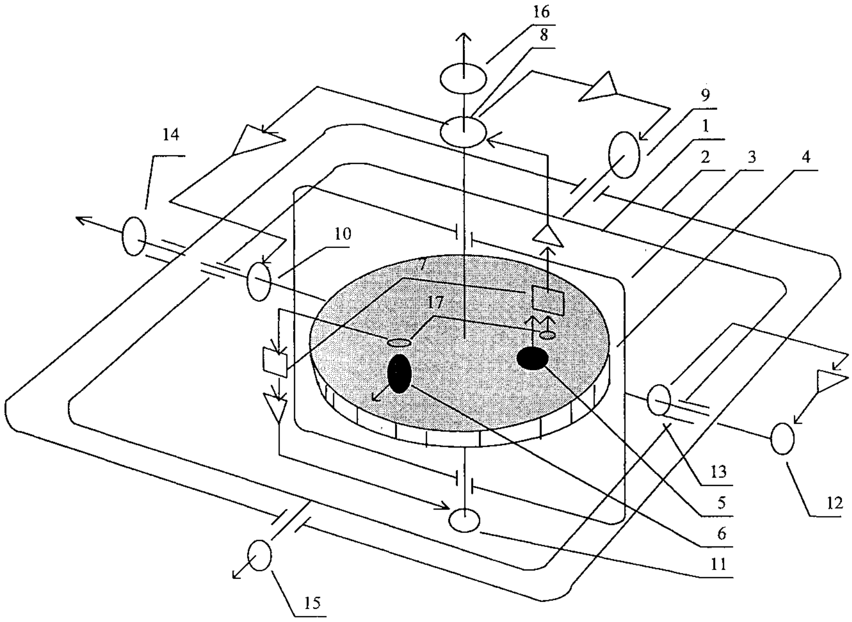

Fig. 1 Kinematics scheme of the three-axis inertial platform stabilization system.

Overshoot:

$$\max|\alpha^k(t)| \le 20'', \qquad k = x, y, z$$

where τ is the interval of the control; $t \in \tau$.

Steady-state error:

$$\lim \alpha^k(t) = 0, \qquad k = x, y, z; \quad t \to \infty$$

Settling time: $t_s \le 0.2$ s, whereas unknown external disturbances are assumed to be step wise constant and have the following values during the simulations:

$$T_L^k = 2 \cdot 10^{-2} \times 1(t - \tau_1)\ \text{Nm} \qquad k = x, y$$

and

$$T_L^z = 1.5 \times 10^{-2} \times 1(t - \tau_2)\ \text{Nm}$$

where τ_1 and τ_2 are the time delays in the application of external disturbances.

Remark 7. These requirements to the stabilization accuracy of the three-axis inertial platform are 2–3 times better then actual performance indexes in the stabilization systems of three-axis inertial platforms with linear controllers placed on some types of Russian wing and ballistic missiles in 1985–1990.

Remark 8. Usually the outputs of the dynamically tuned gyroscope sensors contain noises $y^k = v_g^k + \Delta v_g^k$, $\Delta v_g^k \approx 5 \times \sin(1884t + \Theta_k)$ mV, where 1884 rad/s is the rotational frequency of a rotor of a dynamically tuned gyroscope. The effect of noises of measurements to the control system performance has been reduced by means of the "notch" filters with the transfer functions

$$W_f^k(s) = \frac{\tilde{V}_g^k(s)}{V_g^k(s)} = \frac{1 + 0.28 \cdot 10^{-6}s^2}{1 + 1.41 \cdot 10^{-5}s + 0.28 \cdot 10^{-6}s^2} \qquad k = x, y, z \tag{32}$$

switched on the outputs of the sensors in each axis, where $V_g^k(s)$ and $\tilde{V}_g^k(s)$ are the Laplace transforms of the output voltages of the dynamically tuned gyroscope sensor and the notch filter, respectively. The descriptions of the notch filters (32) have been considered as unmodeled dynamics of the system (30). Nevertheless, transfer functions (32) together with noise description have been employed during the simulations of the three-axis inertial platform sliding mode control system.

Sliding Mode Controller Design in the x Axis of the Three-Axis Inertial Platform

Initially the sliding mode controller design has been performed just in one axis of the three-axis inertial platform. The simplified description [the dynamics of the dynamically tuned gyroscope (29) is neglected] of the x-axis stabilization system has been derived from Eqs. (30) with parameters (31) as

$$\begin{aligned}
\frac{\mathrm{d}i^x}{\mathrm{d}t} &= -20i^x - 13.5\Omega^x + \delta^x(i^x, \Omega^x) + 2.7u^x \\
\frac{\mathrm{d}\Omega^x}{\mathrm{d}t} &= 18.75i^x - 0.7\Omega^x - 250T_L^x \\
\frac{\mathrm{d}v_g^x}{\mathrm{d}t} &= 10^3\Omega^x \\
y^x &= 200v_g^x,
\end{aligned} \tag{33}$$

where y^x is the position angle of the x axis calculated in angular seconds. The new basis for system (33) was introduced following Eqs. (3):

$$\begin{aligned}
z_1 &= i^{x^*} - i^x & z_2 &= \Omega^{x^*} - \Omega^x \\
z_3 &= v_g^{x^*} - v_g^x, & e &= y^{x^*} - y^x
\end{aligned} \tag{34}$$

The equations of the system center (5) (inverse dynamics) has been written in the following form:

$$\begin{aligned}
\frac{\mathrm{d}\Omega^{x^*}}{\mathrm{d}t} &= 18.75i^{x^*} - 0.7\Omega^{x^*} - 250T_L^x \\
\frac{\mathrm{d}v_g^{x^*}}{\mathrm{d}t} &= 10^3\Omega^{x^*} \\
0 &= 200v_g^{x^*} - y^{x^*}
\end{aligned} \tag{35}$$

The unique solution to the differential algebraic equation (35) has been obtained via the method of the differentiation of algebraic constraints.[19] This is

$$i^{x^*} = 13.35 \cdot T_L^x, \qquad \Omega^{x^*} = v_g^{x^*} = 0 \tag{36}$$

No initial conditions have been used to obtain the unique solution (36) of the system of differential algebraic equation (35).

The conventional sliding manifold in the new basis equation (34) has been specified following Eq. (16):

$$\sigma^x = z_1 + 29.3z_2 + 1.33z_3 = 0 \tag{37}$$

The equations of the sliding mode (17) of the system (33) in the new basis equation (34) in the sliding manifold equation (37) have been written as follows:

$$\begin{aligned}
\frac{\mathrm{d}z_2}{\mathrm{d}t} &= -550z_2 - 1.33z_3 \\
\frac{\mathrm{d}z_3}{\mathrm{d}t} &= 10^3z_2 \\
e &= 200z_3
\end{aligned} \tag{38}$$

The discontinuous control law in the form equations (27) and (28), which guarantees the existence of the designed sliding mode, is too complicated for realization. The values of the control functions in expression (2) were selected given physical constraints ($|u^x| \le 30V$) and existence condition[2,3] $\sigma^x(\mathrm{d}\sigma^x/\mathrm{d}t) < 0$ as follows:

$$u^x = U_{\max}^x \cdot \operatorname{sign}(\sigma^x) = 30 \cdot \operatorname{sign}(\sigma^x) \tag{39}$$

to simplify the realization of the sliding mode controller.

The equations of the sliding manifold equation (37) in the initial basis have been changed to the form

$$\sigma^x = 13.35T_L^x - i^x - 29.3\Omega^x - 1.33v_g^x = 0 \tag{40}$$

To realize the control law equations (39) and (40) we assumed i^x, Ω^x, v_g^x to be measured, and the disturbance torque (unmatched function) T_L^x to be estimated by means of the observer

$$\begin{aligned}
\dot{\eta} &= -250\eta + 935i^x + 12500\Omega^x \\
\hat{T}_L^x &= \eta - 50\Omega^x
\end{aligned} \tag{41}$$

The results of the simulation of the x axis stabilization system (29) with the sliding mode controller (39–41), the dynamically tuned gyroscope sensor equation (29), the notch filter equation (32), and measurement noise are shown in Figs. 2 and 3. It is obvious that the specified requirements of the x-axis stabilization system are met.

To avoid the estimation of the disturbance torque T_L^x and the measurement of Ω^x we should acquire the dynamic sliding mode controller. The desired dynamic operator (12) has been specified[20] for system (33) in the new basis equation (34) as follows:

$$H^{x^*}(s) = \frac{s^3 + 248s^2 + 1.33 \cdot 10^5s + 9.23 \cdot 10^6}{1.4 \cdot 10^5s(1 + 1.43s)} \tag{42}$$

The corresponding dynamic operator in the dynamic sliding manifold equation (8) has been designed in the form

$$W(s) = 492.6[(1 + 0.0122s)(1 + 0.0022s)]/s \tag{43}$$

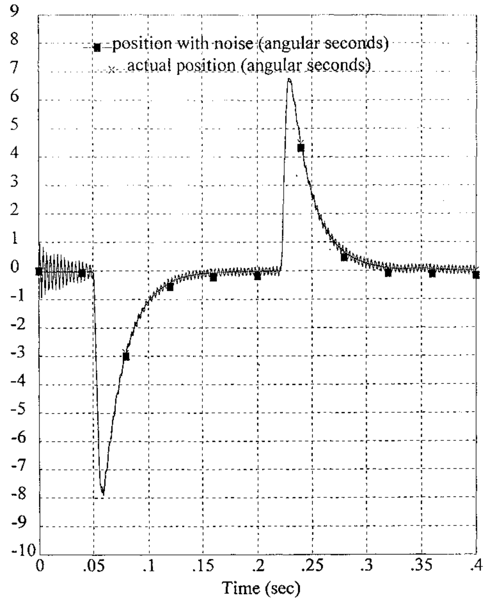

Fig. 2 X-axis conventional sliding mode control, y^x.

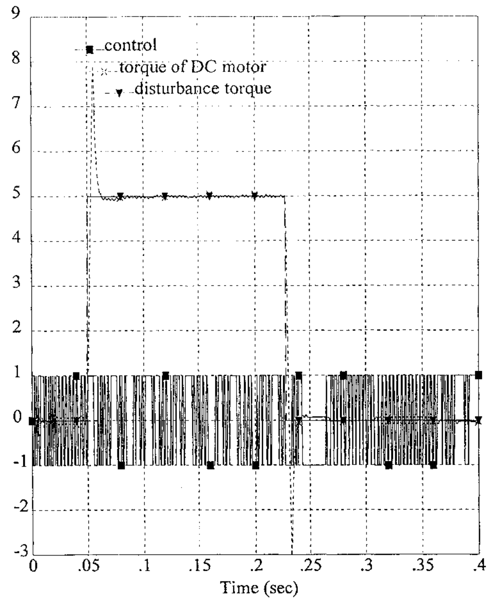

Fig. 3 X-axis conventional sliding mode control, $(u^x/U^x_{\max})$, (T^x_L/J^x), and $(k^x_t \cdot i^x)/J^x$.

The dynamic sliding manifold equation (8) has been formed in the initial basis as

$$\Im^x = -i^x + W(s)e = 0 \tag{44}$$

The values of the control functions in expression (2) were selected regarding given physical constraints ($|u^x| \le 30V$) and existence condition[2,3,12] $\Im^x(\mathrm{d}\Im^x/\mathrm{d}t) < 0$, which implies the inequality

$$U^x_{\max} > |7.4i^x + 5 \cdot [1 - 1.48 \cdot 10^4 \cdot W(s)]\Omega^x - 0.37 \cdot \delta^x(i^x, \Omega^x)|$$

This is

$$u^x = U^x_{\max} \cdot \mathrm{sign}(\Im^x) = 30 \cdot \mathrm{sign}(\Im^x) \tag{45}$$

to simplify the realization of the sliding mode controller.

The results of the simulation of the stabilization system in the x-axis equation (33) of the three-axis inertial platform equation (30) with the dynamic sliding mode controller equations (44) and (45), the dynamically tuned gyroscope sensor equation (29), the notch filter equation (32) and noise of measurement are demonstrated in Figs. 4 and 5. It is obvious that the specified requirements to the stabilization system are met.

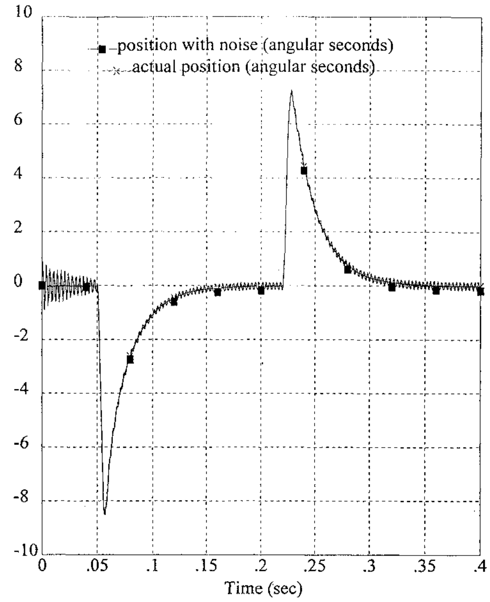

Fig. 4 X-axis dynamic sliding mode control, y^x.

Remark 9. The stabilization system with the dynamic sliding mode controller obtained the combined features of the system with a conventional dynamic compensator (accommodation to unmatched disturbances) and a conventional sliding mode controller (insensitivity to matched disturbances).

Decentralized Sliding Mode Controller Design in the Three-Axis Inertial Platform

To simplify the decentralized sliding mode controller design we will neglect the dynamics of the dynamically tuned gyroscope in the description equation (30) of the three-axis inertial platform. The following simplified description of the three-axis inertial platform stabilization system has been considered:

$$
\begin{gathered}
\frac{\mathrm{d}i^x}{\mathrm{d}t} = -20i^x - 13.5\Omega^x + \delta^x(i^x, \Omega^x) + 2.7u^x \\
\frac{\mathrm{d}\Omega^x}{\mathrm{d}t} = 18.75i^x - 0.7\Omega^x - 2.5 \cdot 10^2 T^x_L + 10^2\Omega^y \\
+ 60\Omega^z - 5 \cdot 10^3 \Omega^z \Omega^y \\
\frac{\mathrm{d}v^x_g}{\mathrm{d}t} = 10^3\Omega^x \\
\frac{\mathrm{d}i^y}{\mathrm{d}t} = -20i^y - 13.5\Omega^y + \delta^y(i^y, \Omega^y) + 2.7u^y \\
\frac{\mathrm{d}\Omega^y}{\mathrm{d}t} = 18.75i^y - 0.7\Omega^x - 2.5 \cdot 10^2 T^y_L + 10^2\Omega^x \\
+ 60\Omega^z - 5 \cdot 10^3 \Omega^z \Omega^x \\
\frac{\mathrm{d}v^y_g}{\mathrm{d}t} = 10^3\Omega^y \\
\frac{\mathrm{d}i^z}{\mathrm{d}t} = -20i^z - 13.5\Omega^z + \delta^z(i^z, \Omega^z) + 2.7u^z \\
\frac{\mathrm{d}\Omega^z}{\mathrm{d}t} = 50i^z - 1.87\Omega^z - 6.67 \cdot 10^2 T^z_L + 2 \cdot 10^2\Omega^y \\
+ 10^2\Omega^x - 10^4\Omega^y\Omega^x \\
\frac{\mathrm{d}v^z_g}{\mathrm{d}t} = 10^3\Omega^z \\
y^x = 2 \cdot 10^2 v^x_g, \qquad y^y = 2 \cdot 10^2 v^y_g, \qquad y^z = 2 \cdot 10^2 v^z_g
\end{gathered}
\tag{46}
$$

where the outputs are measured in angular seconds. The decentralized nonlinear conventional sliding manifolds have been designed concerning expressions (21). They are

$$\sigma^x = \sigma^x_{\text{glob}} + \sigma^x_{\text{loc}}, \qquad \sigma^y = \sigma^y_{\text{glob}} + \sigma^y_{\text{loc}}, \qquad \sigma^z = \sigma^z_{\text{glob}} + \sigma^z_{\text{loc}} \tag{47}$$

The local components of the sliding manifolds in the x, y, and z axes have been designed as follows:

$$\begin{aligned} \sigma^x_{\text{loc}} &= 13.35T^x_L - i^x - 29.3\Omega^x - 1.33v^x_g = 0 \\ \sigma^y_{\text{loc}} &= 13.35T^y_L - i^y - 29.3\Omega^y - 1.33v^y_g = 0 \\ \sigma^z_{\text{loc}} &= 13.35T^z_L - i^z - 11.0\Omega^z - 0.5v^z_g = 0 \end{aligned} \tag{48}$$

to provide the desired output stabilization in each axis in the sliding mode. The global components of the sliding manifolds in x, y, and z have also been designed to decouple the output stabilization errors in the axes in sliding mode. They are

$$\begin{aligned} \sigma^x_{\text{glob}} &= 5.34\Omega^y + 3.2\Omega^z - 267\Omega^y\Omega^z \\ \sigma^y_{\text{glob}} &= 5.34\Omega^x + 3.2\Omega^z - 267\Omega^x\Omega^z \\ \sigma^z_{\text{glob}} &= 4.0\Omega^y + 2.0\Omega^x - 200\Omega^x\Omega^y \end{aligned} \tag{49}$$

Control laws (2) were selected in the form of Eq. (39) as follows:

$$u^k = 30 \cdot \text{sign}(\sigma^k), \qquad k = x, y, z \tag{50}$$

The output stabilization error in each axis in the corresponding sliding manifold is described by decoupled equations (38).

The stabilization system of the three-axis inertial platform equation (46) with the sliding mode controller equations (47–50), the dynamically tuned gyroscope sensor equation (29), and the disturbance observers have been simulated. Nominal load torque has been applied with time delays: 0.05 s in the x axis, 0.15 s in the y axis, and 0.25 s in the z axis.

The results of the system simulation with the omitted global components (49) of the local conventional sliding manifold equations (47) are shown in Figs. 6–8. It is obvious that the specified requirements to the stabilization system are met. However, the effect of the interconnections between the axes is significant.

The results of the system simulation with the decentralized sliding mode controllers are shown in Figs. 9–11. It is obvious that the impact of the interconnections between the axes of the inertial platform is dramatically reduced.

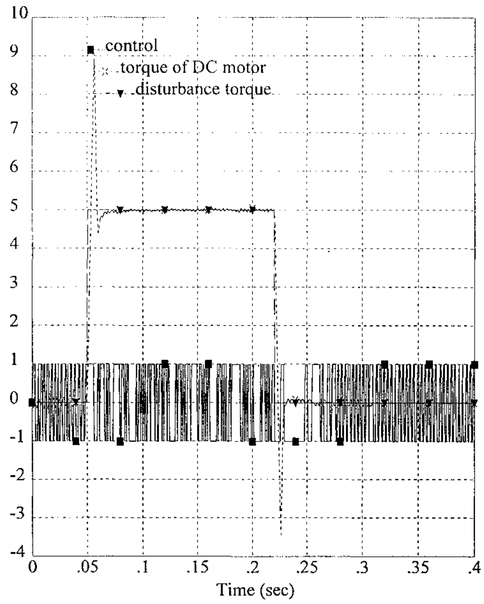

Fig. 5 X-axis dynamic sliding mode control, $u^x/U^x_{\max}$, T^x_L/J^x, and $(k^x_t \cdot i^x)/J^x$.

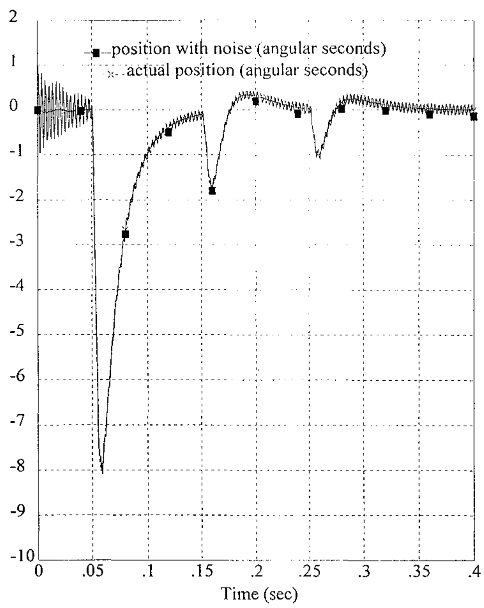

Fig. 6 X-axis conventional sliding mode control with interactions between the axes, y^x.

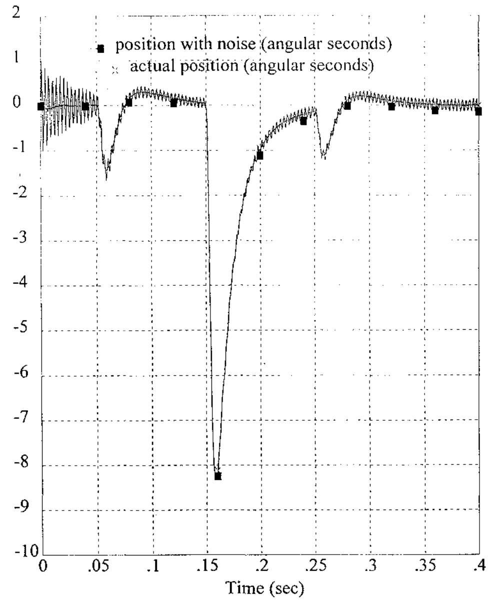

Fig. 7 Y-axis conventional sliding mode control with interactions between the axes, y^y.

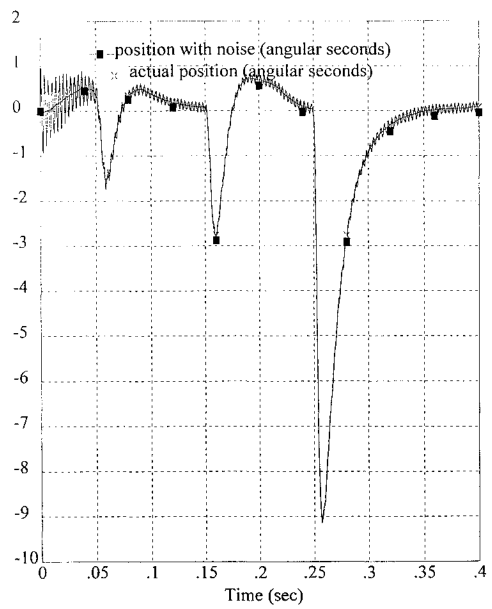

Fig. 8 Z-axis conventional sliding mode control with interactions between the axes, y^z.

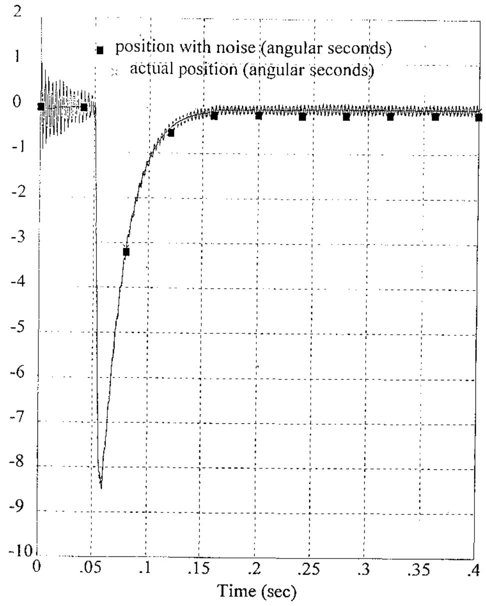

Fig. 9 Decentralized x-axis sliding mode control, y^x.

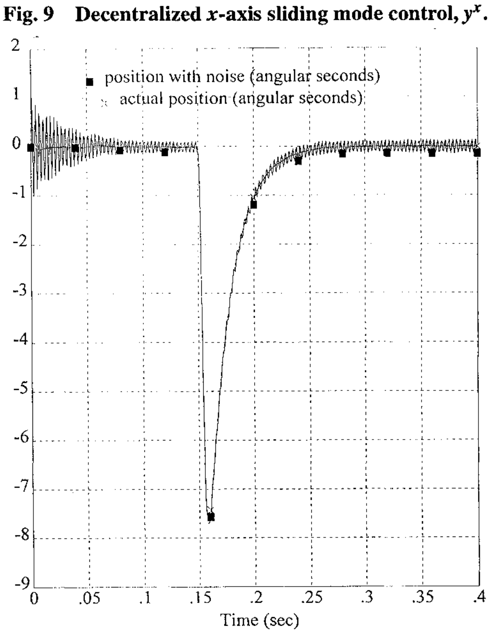

Fig. 10 Decentralized y-axis sliding mode control, y^y.

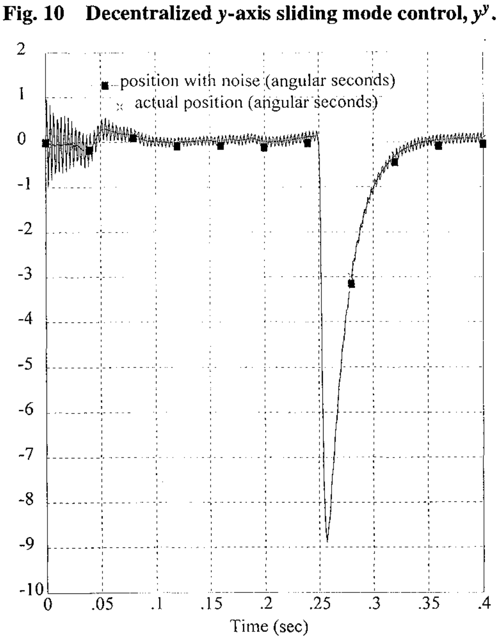

Fig. 11 Decentralized z-axis sliding mode control, y^z.

The decentralized dynamic sliding mode controllers have also been designed. The results of the simulation were pretty much the same as in the system with the conventional decentralized sliding mode controllers, shown earlier. However, the disturbance observers were not employed, and the system demonstrated an accommodation to the external disturbances.

Conclusion

A decentralized sliding mode output tracking (stabilization) strategy, based on a nonlinear conventional and a nonlinear dynamic sliding manifold design, has been devised. The formed sliding mode control algorithms, applied to the design of the stabilization system in three-axis inertial platforms, provided decoupling and high-accuracy stabilization of the outputs in all of the axes of the inertial platform. The inertial platform stabilization system with the dynamic sliding mode controllers obtained the combined features of the system with a conventional dynamic compensator (accommodation to the unmatched disturbances) and a conventional sliding mode controller (insensitivity to the matched disturbances). The designed sliding mode controllers have been realized in inertial platform stabilization systems of some types of Russian wing missiles.

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